

Generalization of the Fourier Sampling Theorem to Irregular Regions of Support

S.K. Nagle and D.N. Levin

Dept. of Radiology, University of Chicago, Chicago, IL, USA, 60637

Introduction: We have developed a superset of the usual Fourier sampling theorem to exploit more general prior knowledge about an object's region of support in order to increase the efficiency of the k -space sampling process. This approach, called "Multiple Region MRI" (mrMRI), enables one to specify the image's region of support as an arbitrary combination of small rectangular "cells" [1,2]. In fact, the conventional sampling theorem emerges as a special case of mrMRI when the supporting cells form a solid rectangular region. If the mrMRI sampling pattern is chosen wisely, the reconstruction may be just as well-conditioned as conventional Fourier reconstruction in the sense that the rms noise in the reconstructed image is the same as the rms noise in the k -space data. In a previous report [1], we derived analytic solutions to the optimal mrMRI sampling patterns for arbitrary combinations of two or three supporting cells. We present here a numerical method for finding the optimal pattern for any arrangement of supporting cells.

Methods: MrMRI is implemented by covering the field of view (FOV) with an array of rectangular cells. K -space is sampled on a small number of sparse, slightly offset, Cartesian grids. Each of these sparse grids has the intersample spacing necessary for Fourier reconstruction of a rectangular FOV with the dimensions of a single cell. The mrMRI image can be reconstructed from the data on all of these sparse grids using a simple linear transformation. During reconstruction noise is amplified by a factor equal to the trace of a matrix that depends only on the relative locations of the supporting cells and the k -space offsets of the sparse grids. This quantity is bounded below by C/A , where C is the number of supporting cells and A is the number of sparse sampling grids making up the mrMRI sampling pattern. If the supporting cell locations are known a priori, the k -space offsets can be chosen to minimize this trace. The resulting mrMRI sampling pattern is optimal in the sense that it minimizes noise amplification during reconstruction. In the special case where the cells coalesce into one rectangular region and $A = C$, the optimal mrMRI pattern reduces to the conventional WKS sampling pattern, and the trace becomes unity as expected.

It is computationally expensive to directly minimize the matrix trace over the space of all possible k -space offsets, especially for large numbers of supporting cells. However, the k -space offsets that minimize the matrix trace also maximize the determinant of the inverse of the same matrix. Because this determinant is a periodic, band-limited function of the k -space offsets, its maximum can be found with less computational effort by performing gradient ascents from a relatively small number of seed points. The set of k -space offsets producing the maximum determinant can then easily be used to calculate the matrix trace and thereby to find the noise amplification expected during mrMRI reconstruction.

We used this strategy to find the optimal mrMRI sampling patterns for a number of different cell geometries. For example, we found the optimal pattern for imaging an object completely con-

tained within a ring-like region containing 4 of the 9 cells of a 3×3 cell array (Fig. 1). This pattern consisted of 4 slightly offset sparse sampling grids. In this case, since we assumed that the object was contained in 4/9 of the rectangular FOV, mrMRI requires only 4/9 of the total number of samples necessary for conventional Fourier reconstruction.

Figure 2 shows a simulated object satisfying this assumption. Simulated k -space data were generated by taking the Fourier transform of this object and adding zero-mean, uncorrelated noise to the resulting k -space values. The simulated k -space data were sampled according to the mrMRI sampling pattern, and the image was reconstructed using mrMRI. The rms noise in the reconstructed image was measured and compared with the rms noise predicted by our trace calculation.

Results: Figure 3 shows the optimal k -space offsets for the 4 sparse grids in the mrMRI sampling pattern. For these offsets, the trace was unity, signifying that the rms noise in the mrMRI reconstruction should equal the rms noise in the simulated k -space data, just as for conventional Fourier image reconstruction. The reconstruction of the simulated object is shown in Figure 4 (windowed to make the noise visible). The rms noise in the reconstructed image was the same as the rms noise in the simulated k -space data, as expected.

We have derived the optimal sampling patterns for a variety of other regions of support, in addition to the example demonstrated here.

Conclusion: We are using the above strategy to develop a library of mrMRI sampling theorems for supporting regions with different geometries. The use of mrMRI will be a simple matter of specifying an image's region of support, looking up the appropriate sampling pattern in the library, sampling the image's k -space according to this pattern, and reconstructing the image with mrMRI. Notice that the computation of optimal sampling patterns requires very little memory or storage space and could easily be parallelized.

MrMRI makes it possible to use prior knowledge of an image's support region to optimize the efficiency of k -space sampling. This increased efficiency can be used to decrease scan time by reducing the required number of k -space measurements. We have taken advantage of this to perform 3D Gd-enhanced carotid MRA with high temporal resolution and with isotropic high spatial resolution without the need for bolus timing. Alternatively, the increased scanning efficiency of mrMRI can be used to reduce image noise or increase the spatial resolution compared to a scan performed in the same time using conventional Fourier sampling.

References:

- [1] Nagle, S.K., Levin, D.N., *Magn. Reson. Med.*, accepted for pub.
- [2] Nagle, S.K., Levin, D.N., Kuperman, V.Yu., *Proc. ISMRM*, 187, 1998.

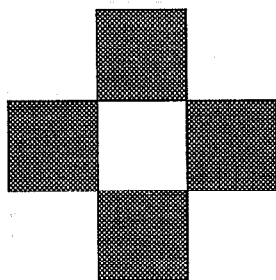


Figure 1

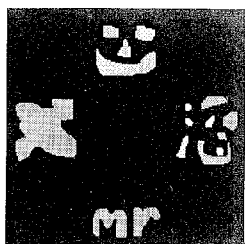


Figure 2

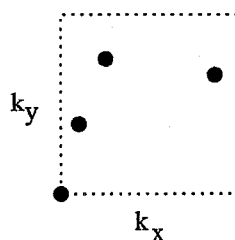


Figure 3

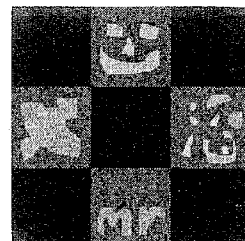


Figure 4